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Na Liu
Student, *orcid.org/0000-0003-4503-3435*
School of Mechanical Engineering, Nanjing University of Science & Technology, Nanjing 243002, PR China
Jiangsu Xindao Machinery Co., Ltd, Yancheng 224005, PR China

MATHEMATICAL MODELING OF HYBRID VEHICLE'S RECUPERATION BRAKING MODE

Abstract. *The paper considers the synthesis of mathematical model of recuperation braking mode for hybrid vehicle as a complex control object. The results of computer simulation as diagrams of transients of different operating parameters of hybrid vehicle power system are obtained on the basis of developed model. The analysis of simulation results confirms the adequacy of the mathematic model of the recuperation braking mode of hybrid vehicle to real processes. The developed model can be used for synthesis of automatic control systems of the electric motors, power converters, power supplies and chargers for hybrid vehicles. Hematical and simulation models of the hybrid vehicle's recuperation braking mode is carried out. The presented models are based on equations of physics of processes and allow to study the recuperation braking mode of the different types hybrid vehicles under various conditions and parameters values (initial linear vehicle's speed, electrical power of generator, inclination angle and the quality of the road surface, etc.). The designed mathematical model has a rather high adequacy to the real processes, which take place in the hybrid vehicles in the recuperation braking mode, that is confirmed by the obtained simulation results in the form of graphs of transients of the main variables changes. Further research should be conducted towards the development of the functional structures, control devices as well as software and hardware for automatic control systems of the different types hybrid vehicles on the basis of the obtained mathematical and simulation models.*

Keywords: hybrid vehicle; mathematic model; recuperation braking mode

Introduction

In recent years, the issue of economic use of world fuel resources is becoming more acute. There are several ways to solve this issue. One of them is the creation of hybrid and electric cars [1-3]. The use of an electric drive with implementation of an energy recovery system makes it possible to replenish energy during the movement of the vehicle. At present, the use of energy recovery systems has reached its greatest development in the field of urban electric and railway transport. Modern research of hybrid cars is mainly related to the modeling of layout schemes, determination of traction and speed properties, energy reserve. The issue of studying the braking properties of hybrid vehicles on which energy recovery and storage systems are applied remains open, as such work is still under development in different countries of the world [4-6].

Complexity of hybrid power systems of modern vehicles as technical objects causes necessity of complex control systems development, that are especially effective at functioning in automatic mode [7-9]. Mathematical and computer modeling methods are appropriate to be used for the research of the efficiency of hybrid engines automatic control system (ACS) at the stage of its development [3, 10, 11]. Analytical mathematic and computer modeling are quite effective and cheap instruments, comparatively with experimental and the other ones. Thus, the development of mathematical model of hybrid engines' is quite necessary.

One of the interesting tasks in the study of hybrid vehicle engines, which deserves special attention, is the research of the recuperation braking mode of a hybrid vehicle [12-14]. Studies of this mode will allow the development of highly efficient energy systems, power converters, power supplies, chargers for hybrid vehicles, as well as their automatic control systems.

The purpose of this work is development and research of mathematical model of hybrid vehicle's recuperation braking mode.

Development of the mathematical and simulation models of hybrid vehicle's recuperation braking mode

The movement of the hybrid vehicle during recuperation braking is described by the equation of force balance for the case of braking a wheeled vehicle [2, 12, 15]:

$$P_i = P_T - P_r - P_n - P_f - P_w, \qquad (1)$$

where P_i is the inertial force of the hybrid vehicle; P_T is the traction force of the main engine of the hybrid vehicle; P_r is the drag force caused by recuperation braking of the hybrid vehicle; P_n is the lift resistance force caused by movement of the hybrid vehicle on an inclined plane; P_f is the rolling resistance force of the hybrid vehicle; P_w is the air resistance force of the hybrid vehicle.

In turn, the inertial force P_i is calculated according to the following equation:

$$P_i = m_{\text{v}} \frac{dv_{\text{v}}}{dt}, \quad (2)$$

where m_{v} is the hybrid vehicle mass; v_{v} is the hybrid vehicle current speed of linear movement.

The traction force of the main engine of the hybrid vehicle P_T is equal to zero in the recuperation braking mode, $P_T = 0$ [2].

The lift resistance force P_n caused by movement of the hybrid vehicle on an inclined plane is defined by the following expression:

$$P_n = m_{\text{v}} g \sin\alpha, \quad (3)$$

where g is the acceleration of gravity; α is the angle of inclination of the plane at which the vehicle is moving.

The rolling resistance force P_f of the hybrid vehicle is calculated using equation (4)

$$P_f = m_{\text{v}} g f \cos\alpha \,\text{sgn}\, v_{\text{v}}, \quad (4)$$

where f is the rolling friction coefficient that depends on the type of road surface on which the vehicle is moving. For hard surfaces, the coefficient f lies within range 0.01…0.03 [2, 12, 13].

In turn, the rolling resistance force P_f always acts in the opposite direction to the movement of the vehicle, which in the equation (4) is described by the sign function from vehicle speed v_{v}.

The air resistance force P_w of the hybrid vehicle is determined by means of the expression (5)

$$P_w = k_{\text{v}} F v_{\text{v}}^2, \quad (5)$$

where F is the drag area of the vehicle; k_{v} is the hybrid vehicle air resistance coefficient.

The drag force P_r caused by recuperation braking of the hybrid vehicle is calculated on the basis of the following equation [3]:

$$P_r = \frac{M_r U_0}{r_n \eta}, \quad (6)$$

where M_r is the generator electromagnetic torque of the recuperation braking; U_0 is the main gear of transmission of the hybrid vehicle; r_n is the wheel radius of the hybrid vehicle; η is the efficiency of the hybrid vehicle generator.

In turn, the generator electromagnetic torque M_r caused by the recuperation braking can be calculated as follows [5]

$$M_r = C_{mM} I_r, \quad (7)$$

where I_r is the current of generator anchor in the recuperation braking mode; C_{mM} is the electromagnetic torque coefficient, which is determined by the parameters of the generator anchor and the value of its magnetic flux.

Moreover, the generator anchor current I_r in the recuperation braking mode is defined by the the electromotive force E_r in the following way

$$E_r = R_{\text{a}} I_r + L_{\text{a}} \frac{dI_r}{dt}, \quad (8)$$

where R_{a} is the total resistance of the anchor winding of the generator and battery charger; L_{a} is the total inductance of the anchor winding of the generator and battery charger.

In turn, the electromotive force E_r of the generator anchor depends on the angular speed of rotation of the generator anchor ω_r in the following way

$$E_r = C_{m\omega} \omega_r, \quad (9)$$

where $C_{m\omega}$ is the electromotive force coefficient, which is determined by the parameters of the generator anchor and the value of its magnetic flux.

The angular speed of rotation of the generator anchor ω_r is connected with the hybrid vehicle current speed v_{v} of linear movement by the formula (10) [6]

$$\omega_r = \frac{v_{\text{v}} U_0}{r_n}. \quad (10)$$

In particular, the hybrid vehicle current speed v_{v} can be determined from equation (2) in the following way

$$v_{\text{v}} = \frac{1}{m_{\text{v}}} \int_{t_1}^{t_2} P_i dt + v_{\text{v}0}, \quad (11)$$

where t_1 and t_2 are selected start and end time moments of the recuperation braking process; $v_{\text{v}0}$ is the initial speed of the vehicle before starting of the recuperation braking process.

To convert vehicle's speed v_{v} in meters per second to kilometers per hour, the first value is multiplied by ratio 3.6.

The electrical power N_{E} of the generator, which is generated during the recuperation braking process, can be calculated using the formula (12)

$$N_{\text{E}} = E_r I_r \cos\varphi, \quad (12)$$

where cosφ is the generator power factor.

In turn, the electrical energy W_{E} generated during the recuperation braking process is found by integrating the electrical power N_{E} of the generator by time

$$W_{\text{E}} = \int_{t_1}^{t_2} N_{\text{E}} dt. \quad (13)$$

Thus, mathematical model of hybrid vehicle's recuperation braking mode consists of the equations (1) – (13). In this paper the study of the recuperation braking mode based on the developed mathematical model is carried out for the hybrid vehicle with the following main parameters: vehicle mass m_{v} = 1200 kg; angle of road inclination α = 0°; rolling friction coefficient f = 0.02; vehicle drag area F = 1.86 m^2; vehicle air resistance coefficient k_{v} = 0.29; main gear of transmission U_0 = 3.875; wheel radius of the vehicle r_n = 0.263 m; nominal electric power of the generator of the hybrid vehicle $N_{Eг}$ = 90 kW; efficiency of the generator η = 0.9; generator power factor cosφ = 0.8; electromagnetic torque coefficient C_{mM} = 1.93; electromotive force coefficient $C_{mω}$ = 1.022; total resistance R_a = 1.72 ohm; total inductance L_a = 0.03 H; initial speed of the vehicle before starting of the recuperation braking process v_{v0} = 100 km/h.

To study the processes of recuperation braking of the hybrid vehicle, in this work, a simulation model is created in Matlab Simulink based on the above equations. The functional structure of the Simulink model of the hybrid vehicle in recuperation braking mode is presented in Fig. 1.

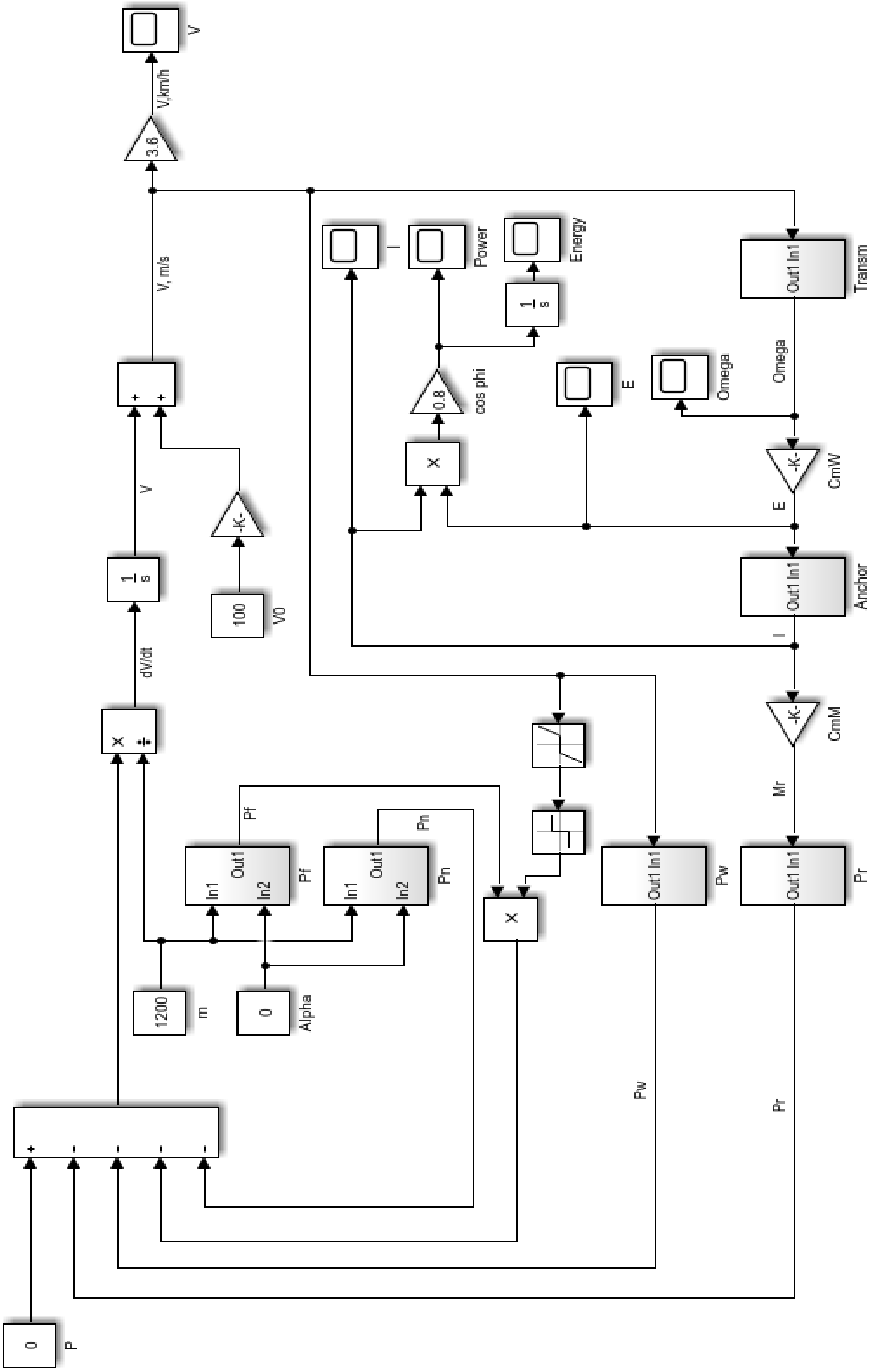

Figure 1 – Functional structure of the Simulink model of the Hybrid Vehicle in Recuperation Braking Mode

The subsystem "Pf" of the rolling resistance force P_f calculation has the structure, presented in Fig. 2.

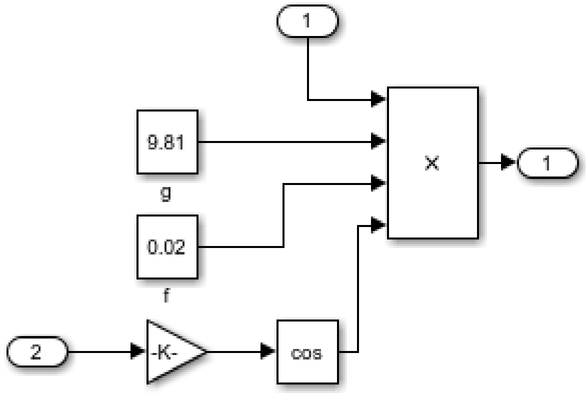

Figure 2 – Structure of the subsystem "Pf" for the rolling resistance force P_f calculation

In turn, the subsystem "Pn" of the lift resistance force P_n calculation has the following structure (Fig. 3)

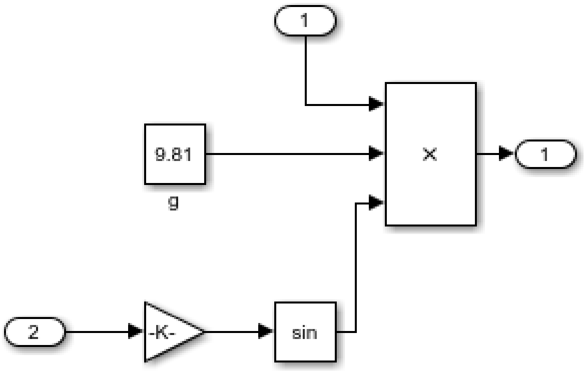

Figure 3 – Structure of the subsystem "Pn" of the lift resistance force Pn calculation

Moreover, the Fig. 4 shows the subsystem "Pw" of the air resistance force P_w calculation.

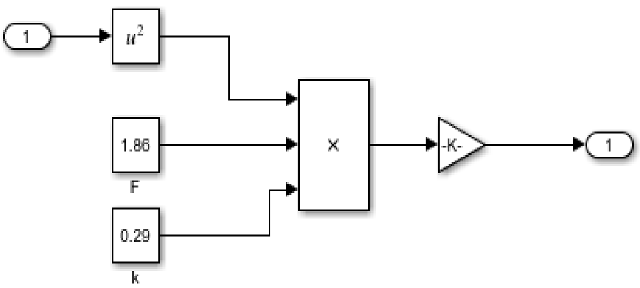

Figure 4 – Structure of the subsystem "Pw" of the air resistance force Pw calculation

The subsystem "Pr" for the calculation of the drag force P_r caused by recuperation braking of the hybrid vehicle is presented in Fig. 5

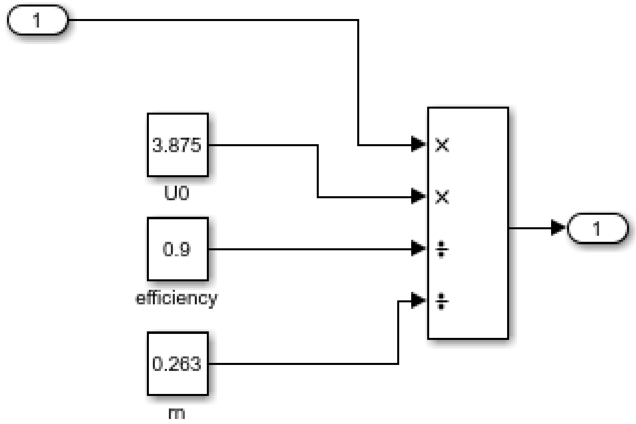

Figure 5 – Structure of the subsystem "Pr" for the calculation of the drag force Pr caused by recuperation braking of the hybrid vehicle

The subsystem "Anchor" for the anchor current I_r calculation based on the electromotive force E_r value has the structure shown in Fig. 6.

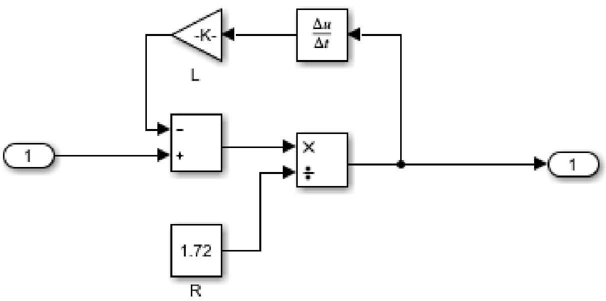

Figure 6 – Structure of the subsystem "Anchor" for the anchor current Ir calculation

Finally, the structure of the subsystem "Transm" of the angular speed of the anchor rotation ω_r calculation is shown in Fig. 7.

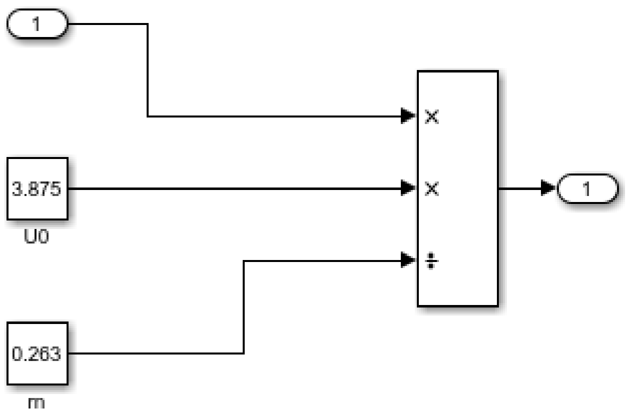

Figure 7 – Structure of the subsystem "Transm" for the angular speed of the anchor rotation ωr calculation

On the basis of the developed simulation model, the hybrid vehicle's recuperation braking mode is simulated with obtaining the following results.

Conclusion

In this work the development and research of mathematical and simulation models of the hybrid vehicle's recuperation braking mode is carried out.

The presented models are based on equations of physics of processes and allow to study the recuperation braking mode of the different types hybrid vehicles under various conditions and parameters values (initial linear vehicle's speed, electrical power of generator, inclination angle and the quality of the road surface, etc.).

The designed mathematical model has a rather high adequacy to the real processes, which take place in the hybrid vehicles in the recuperation braking mode, that is confirmed by the obtained simulation results in the form of graphs of transients of the main variables changes.

Further research should be conducted towards the development of the functional structures, control devices as well as software and hardware for automatic control systems of the different types hybrid vehicles on the basis of the obtained mathematical and simulation models.

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На Лю
Студент, *orcid.org/0000-0003-4503-3435*
Школа машинобудування, Нанкінський університет науки і технологій, Нанкін, КНР
Jiangsu Xindao Machinery Co., Ltd, Yancheng 224005, PR China

МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ РЕЖИМУ ГАЛЬМУВАННЯ ГІБРИДНИХ ТРАНСПОРТНИХ ЗАСОБІВ

Анотація. *Розглянуто синтез математичної моделі режиму рекупераційного гальмування для гібридного автомобіля як складного об'єкта управління. На основі розробленої моделі отримано результати комп'ютерного моделювання як діаграм перехідних процесів різних робочих параметрів гібридної системи живлення транспортного засобу. Аналіз результатів моделювання підтверджує адекватність математичної моделі режиму рекупераційного гальмування гібридного автомобіля реальним процесам. Розроблена модель може бути використана для синтезу систем автоматичного управління електродвигунами, перетворювачами потужності, блоками живлення та зарядними пристроями для гібридних автомобілів. Проведено гематичні та імітаційні моделі режиму гальмування рекуперації гібридного автомобіля. Представлені моделі базуються на рівняннях фізики процесів і дозволяють вивчати режим рекупераційного гальмування гібридних автомобілів різних типів за різних умов та значень параметрів (початкова лінійна швидкість транспортного засобу, електрична потужність генератора, кут нахилу та якість дороги, поверхня тощо). Розроблена математична модель має досить високу адекватність реальним процесам, що відбуваються на гібридних транспортних засобах у режимі рекупераційного гальмування, що підтверджується отриманими результатами моделювання у вигляді графіків перехідних змін основних змінних. Подальші дослідження повинні бути проведені щодо розробки функціональних структур, пристроїв управління, а також програмного та апаратного забезпечення для автоматичних систем управління гібридними автомобілями різних типів на основі отриманих математичних та імітаційних моделей.*

Ключові слова: гібридний транспортний засіб; математична модель; режим рекупераційного гальмування

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